Spur Gear Design

- Pinion
- Gear
Lecture Steps:

1. Quick review, gear geometry (Chapter 8)
2. Transmitted loads (overhead)
3. Review bending stress, bending stress number, \( S_t \), allowable bending stress number, \( S_{at} \) and adjusted allowable bending stress number, \( S'_{at} \).
4. Review contact stress number, \( S_c \), allowable contact stress number, \( S_{ac} \) and adjusted allowable contact stress number \( S'_{ac} \).
5. Overview gear design steps.
6. Example(s) gear design!!
Quick review:

Bending Stress No:

\[ S_t = \left( \frac{(W_t)(P_d)}{(F)(J_p)} \right) (K_oK_sK_mK_BK_v) \]

Required Allowable Bending Stress No:

\[ (S_{at}) = \frac{S_F \times K_R}{Y_{NP}} \times S_t \]

Contact Stress No:

\[ S_c = C_p \sqrt{\left( \frac{W_t \times K_o \times K_s \times K_m \times K_v}{F \times D_p \times I} \right)} \]

Required Allowable Contact Stress No:

\[ (S_{ac}) = \frac{K_R \times S_F}{Z_{NP}} \times S_c \]
Steps for Gear Drive Design:

1. From design requirements, identify speed of pinion, $n_p$, desired output speed of gear, $n_G$, and power to be transmitted, $P$.

2. Choose type of material for the gears (steel, cast iron, bronze, etc.)

3. Determine overload factor, $K_o$, using table 9-5

4. Calculated $P_{des} = K_o P$ and calculate a trial value for the diametral pitch, $P_d$ (for steel use Figure 9-27). Note diametral pitch must be a standard size (see Table 8-2).

   Note, as $P_d$ decreases, tooth size increases thus bringing down $St$ and $Sc$. But..... As $P_d$ increases, # teeth increases and gear train runs smoother and quiter and the drive gets smaller as well!
Steps for Gear Drive Design:

6. Specify $N_p$ and $N_G$ to meet VR requirement. Calculate center distance, $D$, $OD$ to make sure there aren’t any interference issues.

7. Specify face width using recommended range: $8/Pd < F < 16/Pd$.
   - Remember, increasing face width reduces $St$ and $Sc$ but consider alignment factor. Face width is normally less than $2X D_p$.

7. Compute transmitted load, $W_t$, pitch line speed, $v_t$, quality number, $Q_v$, and other factors required for calculating bending stress and contact stress.

8. Calculate $St$ and required $Sat$. Does material in 2 meet $Sat$ #? No – then select new material or define new geometry (step 4). If yes, continue to 9.

9. Calculate $Sc$ and required $Sac$. Does material in 2 meet $Sac$? No – then select new material to meet $Sac$ and $Sat$ or define new geometry (step 4). If yes, continue to 10.

10. Summarize design
Problem # 9.61

A gear pair is to be a part of the drive for a milling machine requiring 20 hp with the pinion speed at 550 rpm and the gear speed to be between 180 and 190 rpm.

**Given:**
- Driven = Milling Machine
- Power = 20 hp
- Pinion Speed = 550 rpm
- Output Speed = 180 - 190 rpm ≈ 185 rpm
- Continuous Use = 30,000 hours

**Find:**
Compact Gear Design

<table>
<thead>
<tr>
<th>Application</th>
<th>Design life (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic appliances</td>
<td>1000–2000</td>
</tr>
<tr>
<td>Aircraft engines</td>
<td>1000–4000</td>
</tr>
<tr>
<td>Automotive</td>
<td>1500–5000</td>
</tr>
<tr>
<td>Agricultural equipment</td>
<td>3000–6000</td>
</tr>
<tr>
<td>Elevators, industrial fans, multipurpose gearing</td>
<td>8000–15 000</td>
</tr>
<tr>
<td>Electric motors, industrial blowers, general</td>
<td>20 000–30 000</td>
</tr>
<tr>
<td>industrial machines</td>
<td></td>
</tr>
<tr>
<td>Pumps and compressors</td>
<td>40 000–60 000</td>
</tr>
<tr>
<td>Critical equipment in continuous 24-h operation</td>
<td>100 000–200 000</td>
</tr>
</tbody>
</table>

**Solution:**

**Design Power:**

Assume: Light Shock Driver and Moderate Shock Driven  
\( K_0 = 1.75 \) (Table 9-5, page 389)

\[
P_{\text{Design}} = (K_0)(P_{\text{Input}}) \\
= (1.75)(20\text{hp}) \\
= 35\text{hp}
\]

**TABLE 9–5**  Suggested overload factors, \( K_0 \)

<table>
<thead>
<tr>
<th>Power source</th>
<th>Uniform</th>
<th>Light shock</th>
<th>Moderate shock</th>
<th>Heavy shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>1.00</td>
<td>1.25</td>
<td>1.50</td>
<td>1.75</td>
</tr>
<tr>
<td>Light shock</td>
<td>1.20</td>
<td>1.40</td>
<td>1.75</td>
<td>2.25</td>
</tr>
<tr>
<td>Moderate shock</td>
<td>1.30</td>
<td>1.70</td>
<td>2.00</td>
<td>2.75</td>
</tr>
</tbody>
</table>
Trial Size
Pick Sizes

$P_d = 5 \text{ T/in}$
$D_p = 4.80 \text{ in}$
$DG = 14.2 \text{ in}$
$N_p = 24 \text{ teeth}$
$NG = 71 \text{ teeth}$

$N_p = P_d \times D_p = (5)(4.80) = 24 \text{ Teeth}$

$VR = \frac{550 \text{ rpm}}{185 \text{ rpm}} = 2.97$

$N_G = (24 \text{ Teeth})(2.97) = 71.28 \text{ Teeth}$

Try $N_G = 71 \text{ Teeth}$

$n_G = \left(\frac{500 \text{ rpm}}{71}\right) = 185.9 \text{ rpm}$

Okay, between $180 - 190 \text{ rpm}$

$D_G = \left(\frac{N_G}{P_d}\right) = \left(\frac{71}{5}\right) = 14.2 \text{ in}$

Check physical size!!
Center Distance

\[ c = \frac{\left( \frac{D_p + D_c}{2} \right)}{2} = \frac{4.80 \text{ in} + 14.2 \text{ in}}{2} = 9.5 \text{ in} \]

Pitch Line Speed

\[ v_z = \frac{\pi D_p n_p}{12} = \frac{(\pi)(4.80 \text{ in})(550 \text{ rpm})}{12} = 691.15 \text{ ft/min} \]

Tangential Load

\[ W_c = \frac{33000 \times \text{Input Horse Power}}{v_z} = \frac{(33000)(20 \text{ hp})}{691.15 \text{ ft/min}} = 955 \# \]

Note: Use Input Power Here as Ko is applied Later!

Face Width

\[ \frac{8}{P_d} < F < \frac{16}{P_d} \]

\[ F = \frac{12}{P_d} = \text{Nominal} = \frac{12}{5} = 2.4 \text{ in} \]
Assumptions:

**Design Decisions**

Quality Number, \( Q_v = 6 \) (Table 9-2, Page 378)

Steel Pinion > Steel Gear

\( C_p = 2300 \) (Table 9-9, Page 400)

More precision, higher quality number!

Softer material, more relative deformation, therefore contact area increases and stress decreases
Geometry Factors

Section 9-8  •  Stresses in Gear Teeth

FIGURE 9–17
Geometry factor, $J$
(Extract from AGMA 218.01 Standard, 
Rating the Pitting 
Resistance and 
Bending Strength of 
Spur and Helical 
Involute Gear Teeth, 
with the permission of 
the publisher, American 
Gear Manufacturers 
Association, 1500 King 
Street, Suite 201, 
Alexandria, VA 22314)

Pinion: $J_p = .36$
Gear: $J_G = .415$

Figure 9-17,
Page 387
Geometry Factors Cont...

\[
\text{Gear Ratio} = \frac{N_G}{N_p} = \frac{71}{24} = 2.96
\]

Page 402

FIGURE 9–23
External spur pinion geometry factor, \( I \), for standard center distances. All curves area for the lowest point of single-tooth contact on the pinion.
(Extracted from AGMA Standard 218.01, 
Rating the Pitting Resistance and Bending Strength of Spur and Helical Involute Gear Teeth, with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)

\[ I = .108 \]

\((a) 20^\circ\) pressure angle, full-depth teeth (standard addendum = \( 1/P_d \))
Load Distribution Factor

\[ K_m = 1 + C_{pf} + C_{ma} \]  \textit{Equation 9-16, Page 390; Equation is solved on next slide}

**Figure 9-18** Pinion proportion factor, \( C_{p} \)

(Extracted from AGMA 2001-C95 Standard, Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth, with permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)

For \( F/D_p < 0.50 \), use curve for \( F/D_p = 0.50 \)

When \( F \leq 1.0 \) in. (\( F \leq 25 \) mm)

\[ C_{pf} = \frac{F}{10D_p} - 0.025 \]

When \( 1.0 \leq F < 15 \),

\[ C_{pf} = \frac{F}{10D_p} - 0.0375 + 0.0125F \]

- Open gearing \( C_{m} = 0.247 + 0.0167F - 0.765 \times 10^{-4} F^2 \)
- Commercial enclosed gear units \( C_{m} = 0.127 + 0.0158F - 1.093 \times 10^{-4} F^2 \)
- Precision enclosed gear units \( C_{m} = 0.0675 + 0.0128F - 0.926 \times 10^{-4} F^2 \)
- Extra-precision enclosed gear units \( C_{m} = 0.0380 + 0.0102F - 0.822 \times 10^{-4} F^2 \)
Load Distribution Factor Cont...

\[ C_{pf} = \frac{F}{(10)(D_p)} - 0.0375 + (0.0125)(F') = \frac{2.4 \text{ in}}{(10)(4.80 \text{ in})} - 0.0375 + (0.0125)(2.4 \text{ in}) = 0.0425 \]

\[ C_{mA} = 0.127 + (0.0158)(F) - (1.093 \times 10^{-4})(F^2) \]
\[ C_{mA} = 0.127 + (0.0158)(2.4 \text{ in}) - (1.093 \times 10^{-4})(2.4 \text{ in}^2) = 0.1643 \]

\[ K_m = 1 + 0.0425 + 0.1643 = 1.2068 \]

Size Factor

\[ k_s = 1.0 \text{ since } P_d \geq 5 \]

<table>
<thead>
<tr>
<th>TABLE 9–6</th>
<th>Suggested size factors, ( K_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diametral pitch, ( P_d )</td>
<td>Metric module, ( m )</td>
</tr>
<tr>
<td>( \geq 5 )</td>
<td>( \leq 5 )</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>1.25</td>
<td>20</td>
</tr>
</tbody>
</table>
Rim Thickness Factor

For this problem, specify a solid gear blank $K_B = 1.00$

$K_B = 1.00$ for $m_B = 1.2$ or larger
Rim Thickness Factor Cont...

\[ h_t = 2a + c \]

\[ a = \frac{1}{P_d} = \frac{1}{5} \]

\[ c = \frac{.25}{P_d} = \frac{.25}{5} = .05 \]

\[ h_t = (2)\left(\frac{1}{5}\right) + (.05) = .45 \text{ in} \]

*We are assuming a solid gear blank for this problem, but if not then use:*

Min Rim Thickness = \((1.2)(.45 \text{ in}) = .54 \text{ in}\)

Min back-up ratio
Safety Factor

\[ S_F = 1.25 \text{ (Mid-Range)} \]

Hardness Ratio

\[ C_H = 1.00 \text{ for early trials until materials have been specified. Then adjust } C_H \text{ if significant differences exist in the hardness of the pinion and the gear.} \]

Reliability

\[ K_R = 1.5 \text{ (for 1 in 10,000 failures)} \]
**Dynamic Factor \( K_v \)**

**Figure 9–21**

Dynamic factor, \( K_v \)

(Extracted from AGMA 2001-C95 standard
Fundamental, Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth.
with the permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)

\[
v_i \text{ max} = \left[ A + (Q_v - 3) \right]^2 \quad \text{(U.S. units)}
\]

\[
v_i \text{ max} = \frac{[A + (Q_v - 3)]^2}{200} \quad \text{(SI units)}
\]

where \( v_i \text{ max} \) = end-point of \( K_v \) curves (ft/min or m/s)

Curves 5–11

\[
K_v = \left( \frac{A + \sqrt{v_i}}{A} \right)^B \quad \text{(U.S. units)}
\]

\[
K_v = \left( \frac{A + \sqrt{200v_i}}{A} \right)^B \quad \text{(SI units)}
\]

where \( A = 50 + 56(1.0 - B) \)

\[
B = \frac{(12 - Q_v)^{0.667}}{4}
\]

\( Q_v \) = transmission accuracy grade number

**Pitch line velocity, m/s**

**Pitch line velocity, \( v_i \), ft/min**

Very accurate gearing

\( Q_v = 5 \quad Q_v = 6 \quad Q_v = 7 \quad Q_v = 8 \quad Q_v = 9 \quad Q_v = 10 \quad Q_v = 11 \)
Dynamic Factor Cont... \( K_v \)

\[
B = \frac{(12 - Q_v)^{6.67}}{4} = \frac{(12 - 6)^{6.67}}{4} = .826
\]

\[
A = 50 + (56)(1 - B) = 50 + (56)(1 - .826) = 59.745
\]

\[
K_v = \left( \frac{A + \sqrt{V_c}}{A} \right)^3 = \left( \frac{59.745 + \sqrt{692.15 \text{ ft/min}}}{59.745} \right)^{.625} = 1.352
\]

\( Q_v \) comes from Figure 9-21

\( K_v \) can be calculated like in above equations or taken from Figure 9-21. Equations are more accurate.
Design Life

\[ N_{cp} = (60)(L)(n)(q) \]
\[ L = 30,000 \text{ hours from Table 9-7} \]
\[ n = 550 \text{ rpm} \]
\[ q = 1 \text{ contacts} \]

\[ N_{cp} = (60)(30,000 \text{ hours})(550 \text{ rpm})(1 \text{ contact}) = 9.9 \times 10^8 \text{ cycles} \]

\[ N_{cg} = (60)(30,000 \text{ hours})(185.92 \text{ rpm})(1 \text{ contact}) = 3.34656 \times 10^8 \text{ cycles} \]
Stress Cycle Factors

\[ Y_{NP} = (1.6831)(N_{cp})^{-0.0323} = (1.6831)(9.9 \times 10^8)^{-0.0323} = 0.862 \]

\[ Y_{NG} = (1.6831)(N_{cg})^{-0.0323} = (1.6831)(3.34656 \times 10^8)^{-0.0323} = 0.893 \]

NOTE: The choice of \( Y_N \) in the shaded area is influenced by the following:
- Pitch line velocity
- Gear material cleanliness
- Residual stress
- Material ductility and fracture toughness

**FIGURE 9–22** Bending strength stress cycle factor, \( Y_N \) (Extracted from AGMA Standard 2001-C95, *Fundamen Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher: American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)
Stress Cycle Factors Cont...

\[ Z_{NP} = (2.466)(N_{cp})^{-0.056} = (2.466)(9.9 \times 10^8)^{-0.056} = 0.773 \]

\[ Z_{NG} = (2.466)(N_{CG})^{-0.056} = (2.466)(3.34656 \times 10^8)^{-0.056} = 0.822 \]

**FIGURE 9-24** Pitting resistance stress cycle factor, \( Z_N \) (Extracted from AGMA Standard 2001-C95, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)
Bending Stress Numbers

Pinion:

\[ S_{tp} = \left( \frac{(W_i)(P_d)}{(F)(J_p)} \right) (K_u K_j K_m K_b K_v) \]

\[ S_{tp} = \left( \frac{(955)(5)}{(2.4)(.36)} \right) (1.75 \times 1.0 \times 1.2068 \times 1.0 \times 1.352) = 15,780.1 \text{ psi} \]

Gear:

\[ S_{tG} = S_{tp} \left( \frac{J_p}{J_G} \right) \]

\[ S_{tG} = 15,780.1 \text{ psi} \left( \frac{.36}{.415} \right) = 13,688.8 \text{ psi} \]
Required Bending Stress Allowable:

\[
(S_{ac})_p = \frac{S_F \times K_R}{Y_{NP}} \times S_{cp} = \left( \frac{(1.25)(1.5)}{0.862} \right) (15,780.1 \text{ pst}) = 34,324.5 \text{ pst}
\]

\[
(S_{ac})_G = \frac{S_F \times K_R}{Y_{NG}} \times S_{cg} = \left( \frac{(1.25)(1.5)}{0.893} \right) (13,688.8 \text{ pst}) = 28,741.9 \text{ pst}
\]
Contact Stress Number

\[ S_c = C_p \sqrt{\left( \frac{W_c \times K_o \times K_z \times K_m \times K_u}{F \times D_p \times I} \right)} \]

\[ S_c = 2300 \sqrt{\frac{(955\#)(1.75)(1.0)(1.2066)(1.352)}{(2.4)(4.80\text{tn})(.108)}} = 107,675 \text{ psi} \]
Required Contact Stress Allowable:

Pinion:
\[
(S_{ac})_p = \frac{K_R \times S_F}{Z_{NP}} \times S_c
\]
\[
(S_{ac})_p = \frac{(1.5)(1.25)}{.773} (107,675 \text{ psi}) - 261,178 \text{ psi}
\]

Gear:
\[
(S_{ac})_G = \frac{K_R \times S_F}{Z_{NG} \times C_H} \times S_c
\]
\[
(S_{ac})_G = \frac{(1.5)(1.25)}{(.822)(1.0)} (107,675 \text{ psi}) = 245,609 \text{ psi}
\]
Hardness Numbers  BENDING (Grade 1)

Pinion Bending
\[ S_{atp} = 34,324.5 \text{ psi} = \text{HB 270} \]

Gear Bending
\[ S_{atG} = 28,741.9 \text{ psi} = \text{HB 215} \]

These stresses are OK

Go to appendix A3 or A4 and spec out material that meets this hardness requirement!  Example  AISI 1040, Temper at 900 F
**Hardness Numbers CONTACT (Grade 1)**

**FIGURE 9-11**
Allowable contact stress number for through-hardened steel gears, $s_{ac}$ (Extracted from AGMA 2001-C95 Standard, *Fundamental Rating Factors and Calculation Methods for Involute Spur and Helical Gear Teeth*, with permission of the publisher, American Gear Manufacturers Association, 1500 King Street, Suite 201, Alexandria, VA 22314)

**Pinion Contact**

$s_{acP} = 261,178$ psi

**Gear Contact**

$s_{acG} = 245,609$ psi

These Stresses are WAY too HIGH! Values are off table!
Summary of Problem

Contact stresses are too High. Must iterate until stress are low enough until a usable material can be found.

NOTE: Contact Stress generally controls. If material cannot be found for bending, contact stress is too high!

Iterate! Decrease $P_d$ and increase $F$

Excel is a GREAT tool to use for these Iterations. This problem solved after third iteration using Excel
Guidelines for Adjustments in Successive Iterations.

1. Decreasing the numerical value of the diametral pitch results in larger teeth and generally lower stresses. Also, the lower value of the pitch usually means a larger face width, which decreases stress and increases surface durability.

2. Increase the diameter of the pinion decreases the transmitted load, generally lowers the stresses and improves surface durability.

3. Increase the face width lowers the stress and improves surface durability but less impact than either the pitch or pitch diameter.

4. Gears with more and smaller teeth tend to run more smoothly and quietly than gears with fewer and larger teeth.

5. Standard values of diametral pitch should be used for ease of manufacture and lower cost (See table 8-2).

6. Use high alloy steels with high surface hardness – results in the most compact system but the cost is higher.

7. Use gears with high quality number, Qv – adds cost but lowers load distribution factor, Km.

8. The number of teeth in the pinion should be as small as possible to make the system compact. But the possibility of interference is greater with fewer teeth. Check Table 8-6 to ensure no interference will occur.