Clutches, Brakes, Coupling, and Flywheels

Clutches, brakes, couplings, and flywheels are a group of elements usually associated with rotation that have in common the function of storing and/or transferring rotating energy.

In analyzing the performance of these devices we shall be interested in:

- The actuating force
- The torque transmitted
- The energy loss
- The temperature rise
Two inertias, $I_1$ and $I_2$, traveling at the respective angular velocities; $\omega_1$ and $\omega_2$, one of which may be zero in the case of brakes, are to be brought to the same speed by engaging the clutch or brake. Slippage occurs because the two elements are running at different speeds and energy is dissipated during actuation, resulting in a temperature rise.

![A simplified dynamic representation of a friction clutch or brake is shown in Fig. 16-la](image)
The various types of devices to be studied may be classified as follows:

- Rim types with internal expanding shoes
- Rim types with external contracting shoes
- Band types
- Disk or axial types
- Cone types
- Miscellaneous types
A flywheel is an inertial energy-storage device. It absorbs mechanical energy by increasing its angular velocity and delivers energy by decreasing its velocity.

Figure 16–lb is a mathematical representation of a flywheel. An input torque $T_i$, corresponding to a coordinate $\theta_i$, will cause the flywheel speed to increase. And a load or output torque $T_o$, with coordinate $\theta_o$, will absorb energy from the flywheel and cause it to slow down.
Static Analysis of Clutches and Brakes

Analyzing general procedure:
• Estimate, model, or measure the pressure distribution on the friction surfaces.
• Find a relationship between the largest pressure and the pressure at any point.
• Use the conditions of static equilibrium to find the braking force or torque and the support reactions.

The net force in the y direction:

\[ N = w_2 \int_0^{w_2} p(u) \, du = p_{av} w_1 w_2 \]

Moment about C from the pressure:

\[ w_2 \int_0^{w_1} p(u) u \, du = \bar{w} w_2 \int_0^{w_1} p(u) \, du = p_{av} w_1 w_2 \bar{u} \]
sum of forces in the x-direction:
\[ \sum F_x = R_x = \pm w_2 \int_0^{w_1} f(p(u) \, du = 0 \]
\[ R_x = \pm w_2 \int_0^{w_1} f(p(u) \, du = \pm f w_1 w_2 \bar{p}_{av} \]

sum of forces in the y-direction:
\[ \sum F_y = -F + w_2 \int_0^{w_1} p(u) \, du + R_y = 0 \]
\[ R_y = F - w_2 \int_0^{w_1} p(u) \, du = F - \bar{p}_{av} w_1 w_2 \]

sum of moment about the pin located at A:
\[ \sum M_A = Fb - w_2 \int_0^{w_1} p(u)(c + u) \, du = af w_2 \int_0^{w_1} p(u) \, du = 0 \]
\[ F = \frac{w_2}{b} \left[ \int_0^{w_1} p(u)(c + u) \, du \pm af \int_0^{w_1} p(u) \, du \right] \]

Can \( F \) be equal to or less than zero?
\[ \int_0^{w_1} p(u)(c + u) \, du - af \int_0^{w_1} p(u) \, du \leq 0 \]

\[ f_{cr} \geq \frac{1}{a} \int_0^{w_1} \frac{p(u)(c + u) \, du}{\int_0^{w_1} p(u) \, du} = \frac{1}{a} \frac{c \int_0^{w_1} p(u) \, du + \int_0^{w_1} p(u)u \, du}{\int_0^{w_1} p(u) \, du} \]
\[ f_{cr} \geq \frac{c + \bar{u}}{a} \]
4.2 Internal Expanding Rim Clutches and Brakes

- Often used in textile machinery, excavators, and machine tools where the clutch may be located within the driving pulley.
- Expanding ring clutches benefit from centrifugal effects; transmit high torque, even at low speeds; and require both positive engagement and ample release force.

- The internal-shoe rim clutch as shown in Fig. 16-3 consists essentially of three elements:
  - The mating frictional surface.
  - The means of transmitting the torque to and from the surfaces.
  - The actuating mechanism.
To analyze an internal-shoe device, figure 16-4 shows a shoe pivoted at point A, with the actuating force acting at the other end of the shoe.

Since the shoe is long, we cannot make the assumption that the distribution of normal forces is uniform.

The mechanical arrangement permits no pressure to be applied at the heel, thus we will assume the pressure at this point to be zero.

Figure 16-4: internal friction shoe geometry
It is the usual practice to omit the friction material for a short distance away from point A. This eliminates interference, and the material would contribute little to the performance anyway, as will be shown.

Let us consider the pressure $p$ acting upon an element of area of the frictional material located at an angle $\theta$ from the hinge pin (Figure 16-4).

We designate the maximum pressure $p_a$ located at an angle $\theta_a$ from the hinge pin.
To find the pressure distribution on the periphery of the internal shoe, consider point B

Deformation perpendicular to AB = hΔϕ.

From the isosceles Δ AOB, h = 2 r sin(ө/2)

\[ h \Delta \phi = 2 r \Delta \phi \sin(\phi/2) = r \Delta \phi \sin(\phi) \]

The deformation, and consequently the pressure, is proportional to \( \sin \theta \).

\[ \frac{p}{\sin \theta} = \frac{p_a}{\sin \theta_a} \]

\[ p = \frac{p_a}{\sin \theta_a} \sin \phi \]

NB

- If the shoe is short, \( p_a \) occurring at the end of the shoe, \( \theta_2 \).
- If the shoe is long, \( p_a \) occurs at \( \theta_a = 90^\circ \)
This pressure distribution has interesting and useful characteristics:

- The pressure distribution is sinusoidal with respect to the angle \( \theta \).
- If the shoe is short, as shown in Fig. 16-6a, the largest pressure on the shoe is \( p_a \) occurring at the end of the shoe, \( \theta_2 \).
- If the shoe is long, as shown in Fig. 16-6b, the largest pressure on the shoe is \( p_a \) occurring at \( \theta_a = 90^0 \).

Figure 16-6: Defining the angle \( \theta_a \) at which the maximum pressure \( p_a \) occurs when
(a) shoe exists in zone \( \theta_1 \leq \theta \leq \pi/2 \)
(b) shoe exists in zone \( \theta_1 \leq \pi/2 \leq \theta_2 \)
When $\theta=0$, Eq. (16-1) shows that the pressure is zero.

- The frictional material located at the heel therefore contributes very little to the braking action and might as well be omitted.

- A good design would concentrate as much frictional material as possible in the neighborhood of the point of maximum pressure.

- In this figure the frictional material begins at an angle $\theta_1$, measured from the hinge pin A, and ends at an angle $\theta_2$. Any arrangement such as this will give a good distribution of the frictional material.

- The actuating force $F$ has components $F_x$ and $F_y$ and operates at distance $c$ from the hinge pin.

Figure 16-7: Forces on the shoe
normal force
\[ p = \frac{p_a}{\sin \theta_a} \sin \theta \]

The moment \( M_f \) of these frictional forces:
\[
M_f = \int f \, dN (r - a \cos \theta) = \frac{f p_a b r}{\sin \theta_a} \int_{\theta_1}^{\theta_2} \sin \theta (r - a \cos \theta) \, d\theta
\]

The moment \( M_N \) of normal force:
\[
M_N = \int dN (a \sin \theta) = \frac{p_a b r a}{\sin \theta_a} \int_{\theta_1}^{\theta_2} \sin^2 \theta \, d\theta
\]

The actuating force \( F \)
\[
M_N = M_f \quad \text{self-locking}
\]

the dimension \( a \) must be such that \( M_N > M_f \)

The torque \( T \):
\[
T = \int f r \, dN = \frac{f p_a b r^2}{\sin \theta_a} \int_{\theta_1}^{\theta_2} \sin \theta \, d\theta
\]

summation of the horizontal and vertical forces:
\[
\begin{align*}
R_x &= \int dN \cos \theta - \int f \, dN \sin \theta - F_x \\
&= \frac{p_a b r}{\sin \theta_a} \left( \int_{\theta_1}^{\theta_2} \sin \theta \cos \theta \, d\theta - f \int_{\theta_1}^{\theta_2} \sin^2 \theta \, d\theta \right) - F_x \\
R_y &= \int dN \sin \theta + \int f \, dN \cos \theta - F_y \\
&= \frac{p_a b r}{\sin \theta_a} \left( \int_{\theta_1}^{\theta_2} \sin^2 \theta \, d\theta + f \int_{\theta_1}^{\theta_2} \sin \theta \cos \theta \, d\theta \right) - F_y
\end{align*}
\]
The following assumptions are implied by the preceding analysis:

- The pressure at any point on the shoe is assumed to be proportional to the distance from the hinge pin, being zero at the heel. This should be considered from the standpoint that pressures specified by manufacturers are averages rather than maxima.
- The effect of centrifugal force has been neglected. In the case of brakes, the shoes are not rotating, and no centrifugal force exists.
- In clutch design, the effect of this force must be considered in writing the equations of static equilibrium.
The shoe is assumed to be rigid. Since this cannot be true, some deflection will occur, depending upon the load, pressure, and stiffness of the shoe.

The resulting pressure distribution may be different from that which has been assumed.

The entire analysis has been based upon a coefficient of friction that does not vary with pressure.

Actually, the coefficient may vary with a number of conditions, including temperature, wear, and environment.
External Contracting Rim Clutches and Brakes

The moments of the frictional and normal forces about the hinge pin are the same as for the internal expanding shoes.

\[ M_f = \frac{f p_a b r}{\sin \theta_a} \int_{\theta_1}^{\theta_2} \sin \theta (r - a \cos \theta) \, d\theta \]

\[ M_N = \frac{p_a b r a}{\sin \theta_a} \int_{\theta_1}^{\theta_2} \sin^2 \theta \, d\theta \]

\[ F = \frac{M_N + M_f}{c} \]

\[ R_x = \int f dN \cos \theta + \int f dN \sin \theta - F_x \]

\[ R_y = \int f dN \cos \theta - \int f dN \sin \theta + F_y \]

Note:
- when external contracting designs are used as clutches, the effect of centrifugal force is to decrease the normal force.
- Thus, as the speed increases, a larger value of the actuating force \( F \) is required.
Δ To get a pressure-distribution relation, we note that lining wear is such as to retain the cylindrical shape.

Δ This means the abscissa component of wear is $w_0$ for all positions $\theta$.

If wear in the radial direction is expressed as $w(\theta)$,

$$w(\theta) = w_0 \cos \theta = KVt$$

As, $w_0/(KVt) = \text{constant}$

$$p(\theta) = (\text{constant}) \cos \theta = p_a \cos \theta$$

**Normal force**

$$dN = pbrd\theta = p_a br\cos(\theta)d\theta$$

The distance $a$ to the pivot is chosen by finding where the moment of the frictional forces $M_f$ is zero.

$$M_f = 2 \int_0^{\theta_2} (f dN)(a \cos \theta - r) = 0$$

Substituting $dN$

$$2fp_a br \int_0^{\theta_2} (a \cos^2 \theta - r \cos \theta) d\theta = 0$$

$$a = \frac{4r \sin \theta_2}{2\theta_2 + \sin 2\theta_2}$$

$K$ = a material constant

$P$ = pressure

$V$ = rim velocity, and

$t$ = time
\[ R_x = 2 \int_0^{\theta_2} dN \cos \theta = \frac{p_{ab}r}{2} (2\theta_2 + \sin 2\theta_2) \]

because of symmetry

\[ \int f \, dN \sin \theta = 0 \]

\[ R_y = 2 \int_0^{\theta_2} f \, dN \cos \theta = \frac{p_{ab}r f}{2} (2\theta_2 + \sin 2\theta_2) \]

\[ \int dN \sin \theta = 0 \]

because of symmetry

\[ R_x = -N \text{ and } R_y = -fN \]

\[ T = afN \]
Band-Type Clutches and Brakes

Because of friction and the rotation of the drum, the actuating force $P_2$ is less than the pin reaction $P_1$

**Summation of force in y-direction:**

$$(P + dP) \sin \frac{d\theta}{2} + P \sin \frac{d\theta}{2} - dN = 0$$

$$dN = Pd\theta$$

**Summation of force in x-direction:**

$$(P + dP) \cos \frac{d\theta}{2} - P \cos \frac{d\theta}{2} - f dN = 0$$

$$dP - f dN = 0$$

$$\int_{P_2}^{P_1} \frac{dP}{P} = f \int_0^\phi d\theta \quad \text{or} \quad \ln \frac{P_1}{P_2} = f \phi$$

Torque

$$T = (P_1 - P_2) \frac{D}{2}$$

Normal force

$$dN = pbr \ d\theta$$

$$P \ d\theta = pbr \ d\theta$$

as $\ dN = p \ d\theta$

$$p = \frac{P}{br} = \frac{2P}{bD}$$

$$p_a = \frac{2P_1}{bD}$$
Machine systems can operate with intermittent motion. Starting and stopping operations can cycle frequently.

Motion control elements permit machine systems to achieve intermittent motion.

**Motion Control elements:**

- **Clutches:** Devices used to transmit power on an intermittent basis by connecting and/or disconnecting a driven component to and/or from the prime mover.
  - Motor operates efficiently at continuous speeds.
  - Avoids accelerating and/or de-accelerating the rotor of the motor each time a driven component of a machine needs to be cycled.

- **Brakes:** Device that absorbs the kinetic energy of a system and thus controls the motion of the system by slowing down the system and/or bringing the system to rest.

**Functions of a clutch:**

- Connect a rapidly turning shaft to one that is initially stationary.
- Cause two shafts to turn at the same speed and to do so in a manner that shock is not produced.
- Limit torque that is transmitted or to prevent torque from being transmitted in a reverse direction.
16.5 Frictional-Contact Axial Clutches

An axial clutch is one in which the mating frictional members are moved in a direction parallel to the shaft.

One of the earliest of these is the cone clutch, which is simple in construction and quite powerful.

Advantages of the disk clutch include

- the more effective heat-dissipation surfaces,
- the favorable pressure distribution.
- the large frictional area that can be installed in a small space,
- the freedom from centrifugal effects,
Let us now determine the capacity of such a clutch or brake in terms of the material and geometry.

Figure 16-16 shows a friction disk having an outside diameter $D$ and an inside diameter $d$.

We are interested in obtaining the axial force $F$ necessary to produce a certain torque $T$ and pressure $p$.

**Two methods of solving the problem, depending upon the construction of the clutch, are in general use.**

If the disks are rigid, then the greatest amount of wear will at first occur in the outer areas, since the work of friction is greater in those areas.

After a certain amount of wear has taken place, the pressure distribution will change so as to permit the wear to be uniform. This is the basis of the first method of solution.

Another method of construction employs springs to obtain a uniform pressure over the area. It is this assumption of uniform pressure that is used in the second method of solution.
Uniform Wear

After initial wear has taken place and the disks have worn down to a point where uniform wear is established, the axial wear can be expressed by:

\[ w = f_1 f_2 K PV t \]

By definition uniform wear is constant from place to place; therefore,

\[ PV = \text{(constant)} = C_1 \]
\[ pr \omega = C_2 \]
\[ pr = C_3 = p_{\text{max}} r_i = p_a r_i = p_a \frac{d}{2} \]

the total normal force

\[ F = \int_{d/2}^{D/2} 2\pi pr \, dr = \pi p_a d \int_{d/2}^{D/2} dr = \frac{\pi p_a d}{2} (D - d) \]

The torque is found by integrating the product of the frictional force and the radius:

\[ T = \int_{d/2}^{D/2} 2\pi f pr^2 \, dr = \pi f p_a d \int_{d/2}^{D/2} r \, dr = \frac{\pi f p_a d}{8} (D^2 - d^2) \]

substituting the value of \( F \)

\[ T = \frac{F f}{4} (D + d) \]
Uniform Pressure

When uniform pressure can be assumed over the area of the disk, the actuating force $F$ is simply the product of the pressure and the area. This gives:

$$F = \frac{\pi p a}{4} (D^2 - d^2)$$

The torque is found by integrating the product of the frictional force and the radius:

$$T = 2\pi f p \int_{d/2}^{D/2} r^2 \, dr = \frac{\pi f p}{12} (D^3 - d^3)$$

Substituting the value of $F$ and noting $p = p_a$

$$T = \frac{F f}{3} \frac{D^3 - d^3}{D^2 - d^2}$$
Disk Brakes

- There is no fundamental difference between a disk clutch and a disk brake.
- The analysis of the preceding section applies to disk brakes too.
- We have seen that rim or drum brakes can be designed for self-energization.
- While this feature is important in reducing the braking effort required, it also has a disadvantage.
- Depicted in Fig. 16-19 is the geometry of an annular-pad brake contact area.

The governing axial wear equation is

$$ w = f_1 f_2 KP V t $$

- $F$ is the coordinate that locates the line of action of force $F$ that intersects the y axis.
- $r_e$ is which is the radius of an equivalent shoe of infinitesimal radial thickness.
- $p$ is the local contact pressure,

the actuating force

$$ F = \int_{\theta_1}^{\theta_2} \int_{r_1}^{r_2} pr dr d\theta = (\theta_2 - \theta_1) \int_{r_1}^{r_2} pr dr $$

the friction torque $T$

$$ T = \int_{\theta_1}^{\theta_2} \int_{r_1}^{r_2} fpr^2 dr d\theta = (\theta_2 - \theta_1) f \int_{r_1}^{r_2} pr^2 dr $$

Figure 16–19 Geometry of contact area of an annular-pad segment of a caliper brake.
The equivalent radius $r_e$ can be found from $ffr_e = T$

The locating coordinate $r\ bar$ of the activating force is found by taking moments about the $x$ axis:

$$M_x = F\bar{r} = \int_{\theta_1}^{\theta_2} \int_{r_i}^{r_o} pr (r \sin \theta) \, dr \, d\theta = (\cos \theta_1 - \cos \theta_2) \int_{r_i}^{r_o} pr^2 \, dr$$

$$\bar{r} = \frac{M_x}{F} = \frac{(\cos \theta_1 - \cos \theta_2)}{\theta_2 - \theta_1} r_e$$

**Uniform Wear**

It is clear that for the axial wear to be the same everywhere, the product $PV$ must be a constant. The pressure $p$ can be expressed in terms of the largest allowable pressure $p_a$ (which occurs at the inner radius $r_i$) as:

$$p = p_ar/r$$

$$F = (\theta_2 - \theta_1)p_ar_i(r_o - r_i)$$

$$T = (\theta_2 - \theta_1)fp_{ar_i} \int_{r_i}^{r_o} r \, dr = \frac{1}{2}(\theta_2 - \theta_1)f_p ar_i (r_o^2 - r_i^2)$$

$$r_e = \frac{\int_{r_i}^{r_o} r \, dr}{p_{ar_i} \int_{r_i}^{r_o} dr} = \frac{r_o^2 - r_i^2}{2} \left( \frac{1}{r_o - r_i} \right) = \frac{r_o + r_i}{2}$$

and

$$\bar{r} = \frac{\cos \theta_1 - \cos \theta_2}{\theta_2 - \theta_1} \frac{r_o + r_i}{2}$$
Uniform Pressure

In this situation, approximated by a new brake, $p = p_a$

\[ F = (\theta_2 - \theta_1) p_a \int_{r_i}^{r_o} r \, dr = \frac{1}{2} (\theta_2 - \theta_1) p_a (r_o^2 - r_i^2) \]

\[ T = (\theta_2 - \theta_1) f p_a \int_{r_i}^{r_o} r^2 \, dr = \frac{1}{3} (\theta_2 - \theta_1) f p_a (r_o^3 - r_i^3) \]

\[ r_e = \frac{p_a \int_{r_i}^{r_o} r^2 \, dr}{p_a \int_{r_i}^{r_o} r \, dr} = \frac{r_o^3 - r_i^3}{\frac{2}{3} \left( \frac{r_o^2}{r_i^2} - 1 \right)} = \frac{2}{3} \frac{r_o^3 - r_i^3}{\frac{2}{3} \left( \frac{r_o^2}{r_i^2} - 1 \right)} \]

\[ \bar{r} = \frac{\cos \theta_1 - \cos \theta_2}{\theta_2 - \theta_1} \frac{2}{3} \frac{r_o^3 - r_i^3}{\frac{2}{3} \left( \frac{r_o^2}{r_i^2} - 1 \right)} \frac{\cos \theta_1 - \cos \theta_2}{\theta_2 - \theta_1} \]
16-7 Cone Clutches and Brakes

- It consists of a **cup keyed or splined to one of the shafts**, a **cone that must slide axially on splines or keys on the mating shaft**, and a **helical spring to hold the clutch in engagement**.
- The clutch is disengaged by means of a fork that fits into the shifting groove on the friction cone.
- The **cone angle** $\alpha$ and the **diameter and face width** of the cone are the important geometric design parameters.
- If the cone angle ($\alpha$) is too small, say, less than about 8°, then the force required to disengage the clutch may be quite large.
- And the wedging effect lessens rapidly when larger cone angles are used.
- Depending upon the characteristics of the friction materials, a good compromise can usually be found using cone angles between 10 and 15°.

![Fig16–21 Cross section of a cone clutch](image)
To find a relation between the operating force $F$ and the torque transmitted, designate the dimensions of the friction cone as shown in Figure 16-22.

As in the case of the axial clutch, we can obtain one set of relations for a uniform-wear and another set for a uniform-pressure assumption.

Element of area $dA = \frac{(2\pi r dr)}{\sin \alpha}$

radius = $r$

width = $dr/\sin \alpha$. 
Uniform Wear

The pressure relation is the same as for the axial clutch:

\[ p = p_a \frac{d}{2r} \]

The operating force will be the integral of the axial component of the differential force \( p \, d \, A \).

\[
F = \int p \, d \, A \sin \alpha = \int_{d/2}^{D/2} \left( p_a \frac{d}{2r} \right) \left( \frac{2\pi r \, dr}{\sin \alpha} \right) (\sin \alpha)
\]

\[
= \pi p_a d \int_{d/2}^{D/2} d\,r = \frac{\pi p_a d}{2} (D - d)
\]

The differential friction force is \( fpdA \), and the torque is the integral of the product of this force with the radius.

\[
T = \int r f p \, d \, A = \int_{d/2}^{D/2} r f \left( p_a \frac{d}{2r} \right) \left( \frac{2\pi r \, dr}{\sin \alpha} \right)
\]

\[
= \frac{\pi f p_a d}{\sin \alpha} \int_{d/2}^{D/2} r \, dr = \frac{\pi f p_a d}{8 \sin \alpha} (D^2 - d^2)
\]
Uniform Pressure

Using \( p = p_a \), the actuating force is found to be

\[
F = \int p_a \, dA \sin \alpha = \int_{d/2}^{D/2} (p_a) \left( \frac{2\pi r \, dr}{\sin \alpha} \right) (\sin \alpha) = \frac{\pi p_a}{4} (D^2 - d^2)
\]

The torque is

\[
T = \int r f p_a \, dA = \int_{d/2}^{D/2} (r f p_a) \left( \frac{2\pi r \, dr}{\sin \alpha} \right) = \frac{\pi f p_a}{12 \sin \alpha} (D^3 - d^3)
\]

Substituting \( F \)

\[
T = \frac{F f}{3 \sin \alpha} \frac{D^3 - d^3}{D^2 - d^2}
\]
Energy Considerations

- When the rotating members of a machine are caused to stop by means of a brake, the kinetic energy of rotation must be absorbed by the brake.
- This energy appears in the brake in the form of heat.
- In the same way, when the members of a machine that are initially at rest are brought up to speed, slipping must occur in the clutch until the driven members have the same speed as the driver. Kinetic energy is absorbed during slippage of either a clutch or a brake, and this energy appears as heat.
- We have seen how the torque capacity of a clutch or brake depends upon the coefficient of friction of the material and upon a safe normal pressure.
- However, the character of the load may be such that, if this torque value is permitted, the clutch or brake may be destroyed by its own generated heat.
- The capacity of a clutch is therefore limited by two factors, the characteristics of the material and the ability of the clutch to dissipate heat.
To get a clear picture of what happens during a simple clutching or braking operation, refer to Fig. 16-la, which is a mathematical model of a two-inertia system connected by a clutch.

As shown, inertias $I_1$ and $I_2$ have initial angular velocities of $\omega_1$ and $\omega_2$, respectively.

During the clutch operation both angular velocities change and eventually become equal.

We assume that the two shafts are rigid and that the clutch torque is constant.

Writing the equation of motion for inertia 1 and 2

$$I_1 \ddot{\theta}_1 = -T \quad \text{and} \quad I_2 \ddot{\theta}_2 = T$$

the instantaneous angular velocities $\dot{\theta}_1$ and $\dot{\theta}_2$

$$\dot{\theta}_1 = -\frac{T}{I_1} + \omega_1$$

$$\dot{\theta}_2 = \frac{T}{I_2} + \omega_2$$

Figure 16–1
(a) Dynamic representation of a clutch or brake; (b) Mathematical representation of a flywheel.
The difference in the velocities, sometimes called the relative velocity,

\[ \dot{\theta} = \dot{\theta}_1 - \dot{\theta}_2 = -\frac{T}{I_1} t + \omega_1 - \left( \frac{T}{I_2} t + \omega_2 \right) \]

\[ = \omega_1 - \omega_2 - T \left( \frac{I_1 + I_2}{I_1 I_2} \right) t \]

If the time required for the entire operation be \( t_1 \) → \( \dot{\theta} = 0 \) → \( \dot{\theta}_1 = \dot{\theta}_2 \)

\[ t_1 = \frac{I_1 I_2 (\omega_1 - \omega_2)}{T (I_1 + I_2)} \]

the time required for the engagement operation is directly proportional to the velocity difference and inversely proportional to the torque

the rate of energy-dissipation during the clutching operation to be

\[ u = T \dot{\theta} = T \left[ \omega_1 - \omega_2 - T \left( \frac{I_1 + I_2}{I_1 I_2} \right) t \right] \]

This shows that the energy-dissipation rate is greatest at the start, when \( t = 0 \).

The total energy dissipated during the clutching operation or braking cycle is

\[ E = \int_0^{t_1} u \, dt = T \int_0^{t_1} \left[ \omega_1 - \omega_2 - T \left( \frac{I_1 + I_2}{I_1 I_2} \right) t \right] dt \]

\[ = \frac{I_1 I_2 (\omega_1 - \omega_2)^2}{2(I_1 + I_2)} \]

Or in US customary units

\[ H = \frac{E}{9336} \]
16-10 Friction Materials

A brake or friction clutch should have the following lining material characteristics to a degree that is dependent on the severity of service:

- High and reproducible coefficient of friction
- Imperviousness to environmental conditions, such as moisture
- The ability to withstand high temperatures, together with good thermal conductivity and diffusivity, as well as high specific heat capacity
- Good resiliency
- High resistance to wear, scoring, and galling
- Compatible with the environment
- Flexibility
**Table 16-2**

Area of Friction Material Required for a Given Average Braking Power  

<table>
<thead>
<tr>
<th>Duty Cycle</th>
<th>Typical Applications</th>
<th>Ratio of Area to Average Braking Power, $\text{in}^2/(\text{Btu/s})$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Band and Drum Brakes</td>
</tr>
<tr>
<td>Infrequent</td>
<td>Emergency brakes</td>
<td>0.85</td>
</tr>
<tr>
<td>Intermittent</td>
<td>Elevators, cranes, and winches</td>
<td>2.8</td>
</tr>
<tr>
<td>Heavy-duty</td>
<td>Excavators, presses</td>
<td>5.6–6.9</td>
</tr>
</tbody>
</table>
### Characteristics of Friction Materials for Brakes and Clutches


<table>
<thead>
<tr>
<th>Material</th>
<th>Friction Coefficient</th>
<th>Maximum Pressure $P_{max}$ psi</th>
<th>Maximum Temperature Instantaneous, °F</th>
<th>Continuous, °F</th>
<th>Maximum Velocity $V_{max}$ ft/min</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comet</td>
<td>0.32</td>
<td>150</td>
<td>1,500</td>
<td>750</td>
<td>3,600</td>
<td>Brakes and clutches</td>
</tr>
<tr>
<td>Sintered metal (dry)</td>
<td>0.29-0.33</td>
<td>300-400</td>
<td>930-1,020</td>
<td>570-660</td>
<td>3,600</td>
<td>Clutches and caliper disk brakes</td>
</tr>
<tr>
<td>Sintered metal (wet)</td>
<td>0.06-0.08</td>
<td>500</td>
<td>930</td>
<td>350</td>
<td>3,600</td>
<td>Clutches</td>
</tr>
<tr>
<td>Rigid molded asbestos (dry)</td>
<td>0.35-0.41</td>
<td>100</td>
<td>660-750</td>
<td>350</td>
<td>3,600</td>
<td>Drum brakes and clutches</td>
</tr>
<tr>
<td>Rigid molded asbestos (wet)</td>
<td>0.06</td>
<td>300</td>
<td>660</td>
<td>350</td>
<td>3,600</td>
<td>Industrial clutches</td>
</tr>
<tr>
<td>Rigid molded asbestos pads</td>
<td>0.31-0.49</td>
<td>750</td>
<td>930-1,380</td>
<td>440-660</td>
<td>4,800</td>
<td>Disk brakes</td>
</tr>
<tr>
<td>Rigid molded non-asbestos</td>
<td>0.33-0.60</td>
<td>100-150</td>
<td>660</td>
<td>300</td>
<td>3,600</td>
<td>Clutches and brakes</td>
</tr>
<tr>
<td>Semi-rigid molded asbestos</td>
<td>0.37-0.41</td>
<td>100</td>
<td>660</td>
<td>300</td>
<td>3,600</td>
<td>Clutches and brakes</td>
</tr>
<tr>
<td>Flexible molded asbestos</td>
<td>0.39-0.43</td>
<td>100</td>
<td>660-750</td>
<td>300-350</td>
<td>3,600</td>
<td>Clutches and brakes</td>
</tr>
<tr>
<td>Wound asbestos yarn and wire</td>
<td>0.38</td>
<td>100</td>
<td>660</td>
<td>300</td>
<td>3,600</td>
<td>Vehicle clutches</td>
</tr>
<tr>
<td>Woven asbestos yarn and wire</td>
<td>0.38</td>
<td>100</td>
<td>500</td>
<td>260</td>
<td>3,600</td>
<td>Industrial clutches and brakes</td>
</tr>
<tr>
<td>Woven cotton</td>
<td>0.47</td>
<td>100</td>
<td>230</td>
<td>170</td>
<td>3,600</td>
<td>Industrial clutches and brakes</td>
</tr>
<tr>
<td>Resilient paper [wet]</td>
<td>0.09-0.15</td>
<td>400</td>
<td>300</td>
<td>$TV &lt; 500,000$ psi · ft/min</td>
<td>3,600</td>
<td>Clutches and transmission bands</td>
</tr>
</tbody>
</table>
Temperature Rise

The temperature rise of the clutch or brake assembly can be approximated by the classic expression

\[ \Delta T = \frac{H}{C_p W} \]  

or

\[ \Delta T = \frac{E}{C_p m} \]

where \( \Delta T \) = temperature rise, °F

\( C_p \) = specific heat capacity, Btu/(lbm \cdot °F); use 0.12 for steel or cast iron

\( W \) = mass of clutch or brake parts, lbm

where \( \Delta T \) = temperature rise, °C

\( C_p \) = specific heat capacity; use 500 J/kg \cdot °C for steel or cast iron

\( m \) = mass of clutch or brake parts, kg
If an object is at initial temperature $T_1$ in an environment of temperature $T_\infty$, then Newton’s cooling model is expressed as

$$\frac{T - T_\infty}{T_1 - T_\infty} = \exp \left( -\frac{h_{CR}A}{WC_p} t \right)$$

where

- $T$ = temperature at time $t$, °F
- $T_1$ = initial temperature, °F
- $T_\infty$ = environmental temperature, °F
- $h_{CR}$ = overall coefficient of heat transfer, Btu/(in$^2 \cdot$ s $\cdot$ °F)
- $A$ = lateral surface area, in$^2$
- $W$ = mass of the object, lbm
- $C_p$ = specific heat capacity of the object, Btu/(lbm $\cdot$ °F)
For repetitive brake applications, subsequent temperature peaks and valleys are recorded

then the rate of heat transfer is described by another Newtonian equation:

\[ H_{\text{loss}} = h_{\text{CR}} A (T - T_\infty) = (h_r + f_v h_c) A (T - T_\infty) \]

where

- \( H_{\text{loss}} \) = rate of energy loss, Btu/s
- \( h_{\text{CR}} \) = overall coefficient of heat transfer, Btu/(in\(^2\) \cdot s \cdot °F)
- \( h_r \) = radiation component of \( h_{\text{CR}} \), Btu/(in\(^2\) \cdot s \cdot °F), Fig. 16–24a
- \( h_c \) = convective component of \( h_{\text{CR}} \), Btu/(in\(^2\) \cdot s \cdot °F), Fig. 16–24a
- \( f_v \) = ventilation factor, Fig. 16–24b
- \( T \) = disk temperature, °F
- \( T_\infty \) = ambient temperature, °F
Example 1: The maximum band interface pressure on the brake shown in the figure is 90 psi. Use a 14-indiameter drum, a band width of 4 in, a coefficient of friction of 0.28, and an angle-of-wrap of 270°. Find the band tensions and the torque capacity.

**Given:** \( D = 300 \text{ mm}, f = 0.28, b = 80 \text{ mm}, \phi = 270^\circ, P_1 = 7600 \text{ N}. \)

**Solution**

\[
f \phi = 0.28(\pi)(270^\circ/180^\circ) = 1.319
\]

\[
P_2 = P_1 \exp(-f \phi) = 7600 \exp(-1.319) = 2032 \text{ N}
\]

\[
p_a = \frac{2P_1}{bD} = \frac{2(7600)}{80(300)} = 0.6333 \text{ N/mm}^2 \text{ or } 633 \text{ kPa} \quad \text{Ans.}
\]

\[
T = \frac{(P_1 - P_2)D}{2} = \frac{(7600 - 2032)300}{2}
\]

\[
= 835200 \text{ N} \cdot \text{mm} \text{ or } 835.2 \text{ N} \cdot \text{m} \quad \text{Ans.}
\]
Example2: A brake has a normal braking torque of 320 N \cdot m and heat-dissipating surfaces whose mass is 18 kg. Suppose a load is brought to rest in 8.3 s from an initial angular speed of 1800 rev/min using the normal braking torque; estimate the temperature rise of the heat-dissipating surfaces.

**Given:** \( T = 320 \text{ N.m} \), \( m = 18 \text{ kg} \), \( t = 8.3 \text{ second} \), \( \omega_1 = 180 \text{ rev/min} \)

**Solution** \( \omega_1 = \frac{2\pi n}{60} = \frac{2\pi(1800)}{60} = 188.5 \text{ rad/s} \) and \( \omega_2 = 0 \)

the time required for engagement operation:

\[
t_1 = \frac{I_1 I_2 (\omega_1 - \omega_2)}{T (I_1 + I_2)}
\]

The total energy dissipated during the clutching operation or braking cycle is

\[
E = \int_0^{t_1} u \, dt = T \int_0^{t_1} \left[ \omega_1 - \omega_2 - T \left( \frac{I_1 + I_2}{I_1 I_2} \right) t \right] \, dt
\]

\[
= \frac{I_1 I_2 (\omega_1 - \omega_2)^2}{2(I_1 + I_2)}
\]

\[
\Delta T = \frac{E}{C_p m} = \frac{250(10^3)}{500(18)} = 27.8 \text{^\circ C} \quad \text{Ans.}
\]