3. The flow mechanism in the rotor of a turbomachine
Designations of velocities:

- **U - peripheral (angular) velocity**
- **C – absolute velocity:** velocity of the flow when viewed from a stationary frame of reference.
- **W - relative velocity**:-velocity of the flow when viewed from a rotating component frame of reference

Designation of angles:

- **α:** Angle between **U** and **C**
- **β:** angle between **W** and negative direction of **U**
The vane congruent Flow

Example: radial-flow blower

suction end

meridian section

relative path = course of vane

velocity triangles

absolute path
Vane congruent flow: Streamlines of the flow are congruent to the vanes. This assumption holds:

- If there are infinite numbers of vanes
- If the vanes are infinitely thin

If there exist *such a deviation between the direction of the flow and the direction of the vane at the entrance of the vane channel*, the flow is said to approach the vane with *‘Shock’*. 

\[ \beta \]

The Actual Flow Pattern
Fundamental Equation of Turbomachinery

- From Newton’s second law of Motion: ‘Torque is equal to the rate of change of angular momentum’.

\[ T = \frac{dL}{dt} \]

Where T is torque and L is angular momentum.
• The angular momentum becomes: \[ L = mV_\theta r \]

• At entry to the turbomachinery the angular momentum is equal to \( m_1 V_\theta_1 r \) and at the exit it becomes: \( m_2 V_\theta_2 r \).

• The change in angular momentum becomes: \[ m_2 V_\theta_2 r - m_1 V_\theta_1 r \]

• And the time rate of change is given as:

  \[
  \frac{m_2 V_\theta_2 r_2 - m_1 V_\theta_1 r_1}{t} = \dot{m}_2 V_\theta_2 r_2 - \dot{m}_1 V_\theta_1 r_1 = \dot{m}(V_\theta_2 r_2 - V_\theta_1 r_1)
  \]

• Since “what goes in must come out”:

  \[ \dot{m}_2 = \dot{m}_1 = \dot{m} \]
Since "the time rate of angular momentum is equal to the torque" we have:

\[ T = \dot{m}(V_{\theta 2} r_2 - V_{\theta 1} r_1) \quad \text{or} \quad T_{\text{blade}} = \rho V(r_2 C_{3U} - r_1 C_{0U}) \]
• The power transferred between the rotor vanes/blades and the flow follows from the blade torque:

$$N_{\text{blade}} = \omega T_{\text{blade}} = Y_{\text{blade}} \rho V$$

• Thus, the specific work $Y_{\text{blade}}$ done by the vanes/blades follows:

$$Y_{\text{blade}} = \frac{N_{\text{blade}}}{\rho V} = \frac{\omega T_{\text{blade}}}{\rho V} = \omega \left( r_2 C_{3U} - r_1 C_{0U} \right) \quad \text{or} \quad Y_{\text{blade}} = U_2 C_{3U} - U_1 C_{0U}$$

• The above equation applies for both pumps and turbines.

• Often the flow at the suction side has no vortex: $r_1 C_{ou} = 0$;

$$Y_{\text{blade}} = U_2 C_{3U}$$
• \( Y_{blade} \text{ is independent of } \rho. \)

• The independence of \( Y \) from \( \rho \) has a \textit{considerable influence on the pressure difference between suction and pressure ends} of the machine if the same machine is used for flow media with different \( \rho \).

• Example: pumping machine with \( V = 0.1 \text{m}^3/\text{kg} \) and \( Y = 1000 \text{ J/Kg} \)

<table>
<thead>
<tr>
<th>Water pumping</th>
<th>Air pumping</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho = 1000 \text{ kg/m}^3 )</td>
<td>( \rho = 1.2 \text{ kg/m}^3 )</td>
</tr>
<tr>
<td>Pressure rise: ( P_D - P_S = \rho Y = 10^6 \text{N/m}^2 = 10 \text{ bar} )</td>
<td>( P_D - P_S = \rho Y = 1200 \text{N/m}^2 = 0.012 \text{bar} )</td>
</tr>
<tr>
<td>( \text{Neff} = \rho V Y = 10^5 \text{ W} = 100 \text{kW} )</td>
<td>( \text{Neff} = \rho V Y = 120 \text{ W} = 0.12 \text{kW} )</td>
</tr>
</tbody>
</table>
• For the above example for a given spec. Work Y is the pressure rise is very small for air/gas pumping compared with water pumping.

• High values of spec. work are needed for air/gas pumping. Thus higher values of U are required. (U_{max} = 300 to 350 to 450 m/s).

• Hence, radial-flow impellers for water/liquid pumping which does not need high values of Y can be build of cast iron or brass.
• But impellers of radial-flow air/gas compressors which need very high \( Y \) and thus high \( U \), are built of forged disk, which are thickened towards the center considerably according to the higher strain there.

To obtain high pressure difference several rotors are arranged in series.
The velocity Triangle

- Pressure side of rotor (reference point 2)
  - $C_{2m} = W_{2m}$
  - $c_2$

- Suction side of rotor (reference point 1)
  - $C_{1m} = W_{1m}$
  - $c_1$

- $\beta_1$
- $\alpha_1$
- $\beta_2$
- $\alpha_2$
- $u_2$
- $u_1$
- $W_{2u}$
- $W_{1u}$
- Pump
- Turbine
• The absolute velocity $C$ of a point on the rotating rotor is given by the **vectorial sum of the relative velocity** $W$ **and the peripheral velocity** $U$ of the point under consideration.

• $C$ can be determined graphically using the velocity parallelogram. The method may be simplified by using only one of the triangles of the parallelogram, the so called "**Velocity Triangle**."

• $C_{IU}$ and $C_{2U}$ are measures of the specific work $Y$;

• $C_{Im}$ and $C_{2m}$ are measures of the volume $V$. 
Relationship b/n flow rate and the geometry of the machine

\[ V_o = (2\pi r_1 b_1)C_{or} \quad \text{Volume flow rate at 0} \]
\[ V_3 = (2\pi r_3 b_3)C_{3r} \quad \text{Volume flow rate at 3} \]

- Where \( b_1 \) and \( b_2 \) are blade width at point 1 and 2
- The mass flow rate can be calculated from

\[ M_0 = M_3 = M \quad \text{From continuity eqn.} \]
\[ M = \rho_0 V_0 = \rho_3 V_3 \]

- If the density \( \rho \) is constant:

\[ V = 2\pi r_1 b_1 C_{or} = 2\pi r_3 b_3 C_{3r} \]
Determination of the Vane Angles $\beta_1$ and $\beta_2$
• The velocity triangles for the points 0 and 3 can be drawn if the following are known:
  – Peripheral velocities,
  – The meridian velocities and
  – The flow rotation at the suction side of the rotor
• If the average flow line followed the curvature of the vane and if the vane were infinitely thin, the direction of the flow line at points 0 and 3 would coincide with the vane curvature at points 1 and 2.

• In this case it would be $\beta_1 = \beta_0$ and $\beta_2 = \beta_3$ and the vane ends at would be determined.

• But the average low line sustains deviations unless the vanes are infinitely thin and infinitely close to each other.

• Both conditions cannot be fulfilled, the first one as the strength of the material demands a certain thickness, the second one as otherwise no flow could pass through the vane channel.
Influence of the Definite Thickness of the vanes

- \( Z \) = number of vanes
- \( t \) = thickness of vanes
- \( S = \pi D/Z \) pitch
- \( \sigma = t / \sin \beta \) peripheral thickness (per. To \( C_m \))
- \( b \) = width of channel

\[ \beta'_2 = \text{vane angle if only the influence of thickness is taken in account} \]
• Applying the equation of continuity for points 0 and 1:

\[
V_{\text{channel}} = b_1 S_1 c_{0m} = b_1 (S_1 - \sigma_1) c_{1m}
\]

thus, \( c_{1m} = c_{0m} \frac{S_1}{S_1 - \sigma_1} \), it follows that \( c_{1m} > c_{0m} \) and \( \beta_1 > \beta_0 \)

From the equation of continuity follows

\[
c_{2m} = c_{3m} \frac{S_2}{S_2 - \sigma_2}
\]
hence, \( c_{2m} > c_{3m} \)

\( \beta'_2 > \beta_3 \)

• The factor \( \frac{S}{S - \sigma} \) may be called “Vane contraction factor”

<table>
<thead>
<tr>
<th></th>
<th>Radial Flow</th>
<th>Axial Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_1/(S_1 - \sigma_1) )</td>
<td>1.1 to 1.2</td>
<td>1.04 to 1.06</td>
</tr>
<tr>
<td>( S_2/(S_2 - \sigma_2) )</td>
<td>1.01 to 1.03</td>
<td>1.01 to 1.03</td>
</tr>
</tbody>
</table>
Velocity triangles in axial Flow Machines

Figure 1.2: A Simple Turbine
Figure 1.3: A Simple Turbine: Exploded View
Figure 1.4: Simple Turbine Operation

1. Flow arrives in the axial direction
2. Flow is turned in the stator blades
3. Flow is turned back by the rotor blades
• The cascade view arises from looking at the stator and rotor of a turbomachine closely.
For the meridional view instead of looking at the tip of the blade this time we take a side on view of the whole turbine and look at a cross section of the machine at the hub and tip radius.
Figure 2.4: Cascade and Meridional Views of a Turbine Stage
Figure 2.5: Velocity Triangles for a Turbine Stage