The specific Static Rotor Work Yp

Specific Static rotor work

$$Y_P = \frac{1}{\rho}\left(P_3 - P_0\right)$$

Where P_0, P_3 = static pressures at points 0,3

$(P_0 - P_3)$ = static pressure difference of the rotor

ρ = density, in case of a compressible medium average of ρ_3 and ρ_3 .

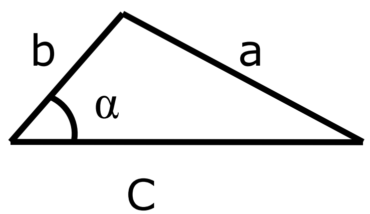
Yp can be calculated from the energy difference of the flow medium between o and 3

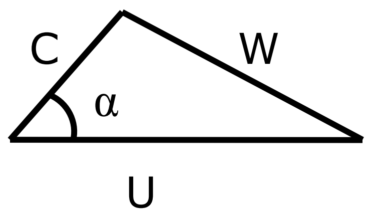
$$Y_{blade} \mp Z_u = \frac{P_3 - P_0}{\rho} + \frac{C_3^2 - C_0^2}{2}$$

Where

$$Y_{blade} = U_2 C_{3U} - U_1 C_{0U} = U_2 C_3 \cos\alpha_3 - U_1 C_0 \cos\alpha_0$$

- Applying the cos-theorem of a triangle





$$2bc\cos\alpha = b^2 + c^2 - a^2 \longrightarrow 2CU\cos\alpha = C^2 + U^2 - W^2$$

$$\begin{aligned} Y_{blade} &= \frac{1}{2}\left(C_3^2 + U_3^2 - W_3^2 - C_0^2 - U_0^2 + W_0^2\right) \\ &= \frac{1}{2}\left(C_3^2 - C_0^2 + U_3^2 - U_0^2 + W_0^2 - W_3^2\right) \end{aligned}$$

- It follows

$$Y_P = Y_{blade} - \frac{C_3^2 - C_0^2}{2} \mp Z_u = \frac{1}{2}\left(U_3^2 - U_0^2 + W_0^2 - W_3^2\right) \mp Z_u$$

Bernoulli Equation of the Relative Flow

- Neglecting the hydraulic loss, i.e. $Z_u = 0$,

$$Y_P = \frac{P_3 - P_0}{\rho} = \left(\frac{W_0^2}{2} - \frac{U_1^2}{2}\right) - \left(\frac{W_3^2}{2} - \frac{U_2^2}{2}\right)$$

- It follows

$$\frac{P_3}{\rho} + \frac{W_3^2}{2} - \frac{U_2^2}{2} = \frac{P_0}{\rho} + \frac{W_0^2}{2} - \frac{U_1^2}{2}$$

- The above formula applies to any points along the flow line passing the vane channel

$$\frac{P}{\rho} + \frac{W^2}{2} - \frac{U^2}{2} = const$$

Bernoulli Equation of the Relative Flow

Impulse and Reaction Type of Turbomachines

- Considering Y_P, the turbomachine can be grouped into:

A. Turbomachines without pressure difference in front of and beyond the rotor, i.e $(P_3 - P_0) = 0$ or $Y_p = 0$ ***"Impulse" type of Turbomachines***

B. Turbomachines with pressure difference in front and beyond the rotor, i.e. $(P_3 - P_0) \neq 0$ $Y_p > 0$ ***Reaction type of Turbomachines***

❖For impulse turbines

- The total head of the incoming fluid is converted into a large velocity head at the exit of the supply nozzle.
- Both the pressure drop across the bucket (blade) and the change in relative speed of the fluid across the bucket are negligible.
- The space surrounding the rotor is not completely filled with fluid.
- The individual jets of fluid striking the buckets that generates the torque.

❖For reaction turbines

- There is both a pressure drop and a fluid relative speed change across the rotor.
- Guide vanes act as nozzle to accelerate the flow and turn it in the appropriate direction as the fluid enters the rotor.
- Part of the pressure drop occurs across the guide vanes and part occurs across the rotor,

❖Summary

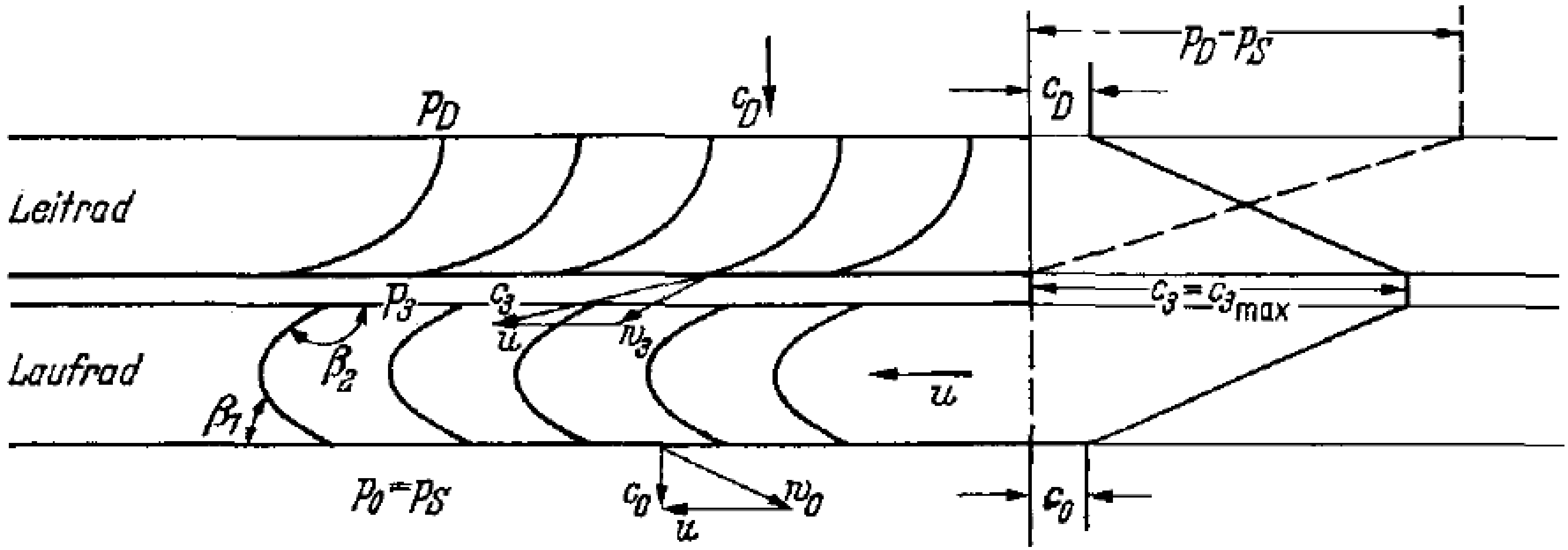
– **Impulse turbines: High-head, low flowrate devices.**

– **Reaction turbines: Low-head, high-flowrate devices.**

Equal Pressure or Impulse Type of Turbomachines

$$P_3 - P_0 = 0 \; and \; Y_P = 0$$

- Example a. Single-Stage Steam Turbine



- The entirely available *pressure difference (P_3-P_0) is converted into velocity* while the flow passes through the *stationary guide vanes*

- The velocity existing in the clearance between the stationary guide vanes and the rotor blades is the highest one which can be obtained from the available pressure difference, i.e. $C_3 = C_{3max\ attainable}$

- The *kinetic energy of the flow entering the rotor is reduced while the flow passes the blade channels*, *the absolute velocity is reduced from C_3 to C_0*.

- The specific static rotor work Yp is (for axial flow $U_1 = U_2 = U$)

$$Y_P = \frac{1}{2}\left(W_0^2 - W_3^2\right) + Z_u$$

- Neglecting the hydraulic lose Z_u of the rotor, it follows because $Y_p = 0$.

$$W_0 = W_3$$

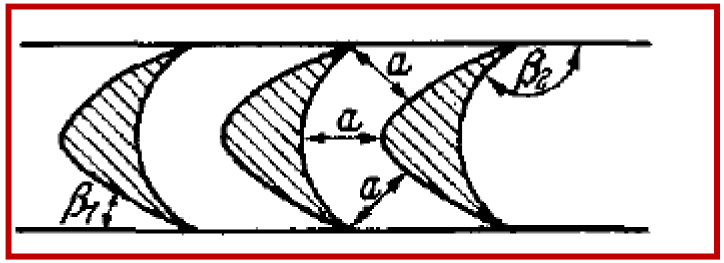
- Considering the loss:

$$W_0 = \varphi W_3$$

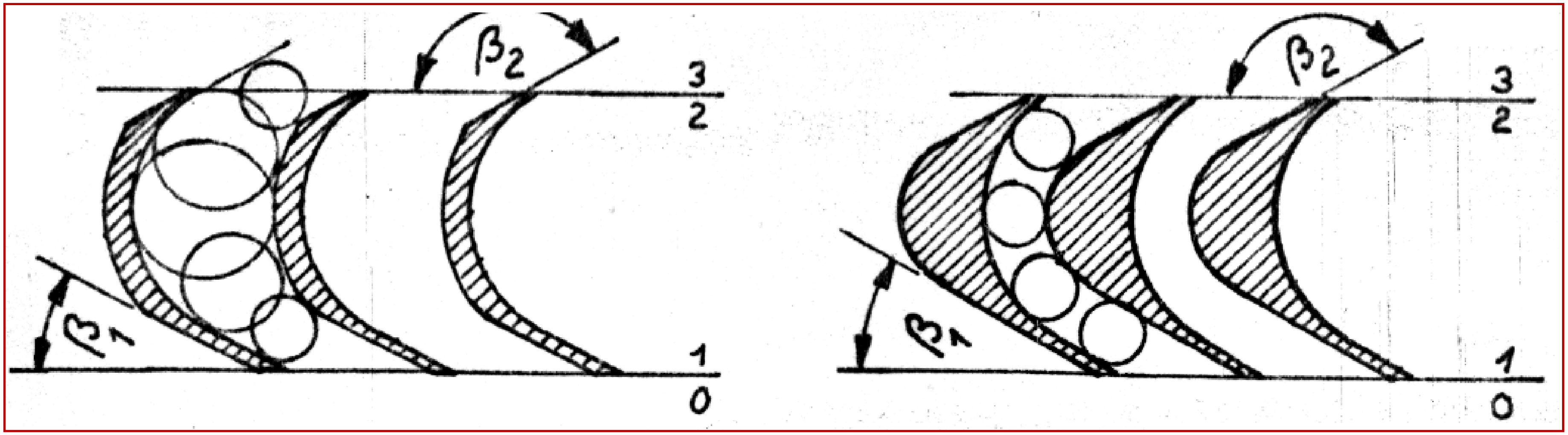
- Where the velocity coefficient ϕ takes in to account the drop in kinetic energy due to Z_u; $\Phi<1$.

- The condition $W_o \approx W_3$ demands rotor blades of the 'hook-form' type, i.e. $\beta_2 > 90^0$.

Blades of a constant-pressure steam or Gas turbine. 'a' is the channel width at all points approximately equal



- If blade has uniform thickness, the flow while passing the channel is first decelerated then accelerated.
- Such change in the flow velocity is undesirable as it leads to *unnecessary losses.*
- In order to obtain W≈ const. along the vane channel the blade be designed with strong profiling; however, such blades are costly



- The specific work Y_{blade} of an impulse steam turbine stage as for a given velocity U_2 proportional to the velocity C_3

$$Y_{blade} = U_2 C_{3U} = U_2 C_3 \cos\alpha_3 \propto C_3 = C_{3\max-att.}$$

For $\alpha_0 = 90^0$

- Steam turbines are designed with approximately the same angle α_3=15 to 20 degrees.

- As C_3 of impulse team turbines has highest possible value $C_{3max\text{-}att.}$ The spec. work Y_{blade} of these turbines has highest value

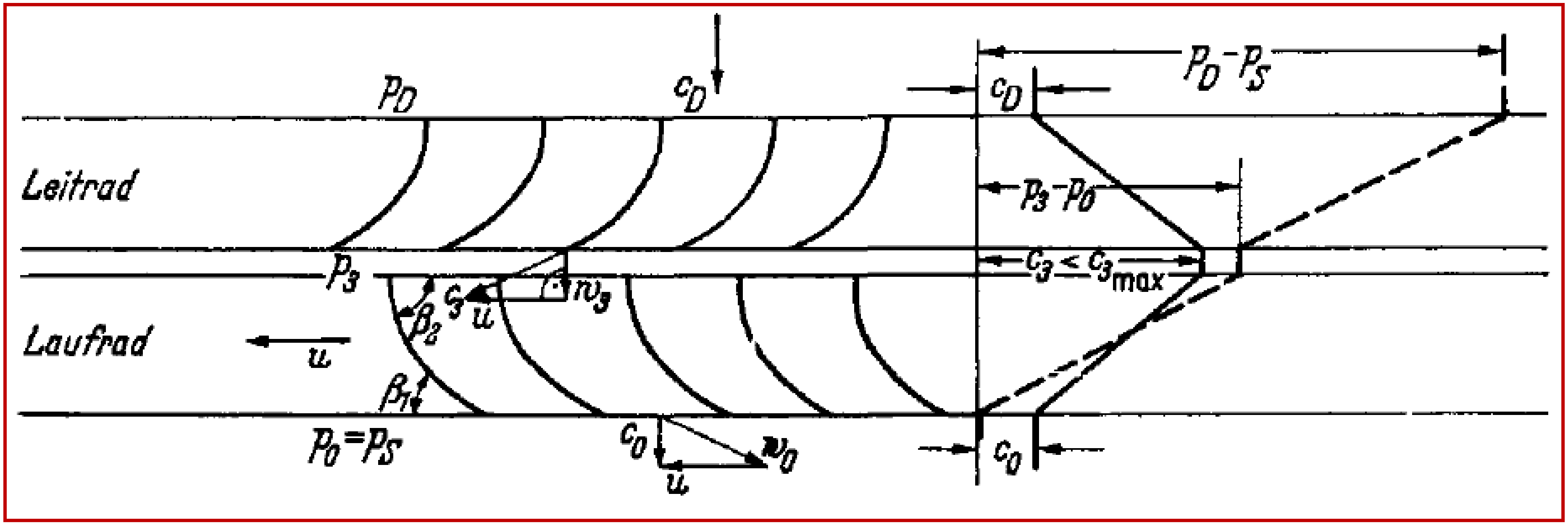
$$Y_{blade-impulse\ t.} = Y_{blade-\max.att.} \qquad \textit{for a given } U_2$$

- *The peripheral velocity U_2 will be lowest for a given Y_{blade} if the turbine is designed as impulse turbine*

- ***Impulse turbines are slow running turbines***

Over-Pressure or Reaction Type of Turbomachine

- Example: Single-Stage Reaction Steam Turbine



- While the flow passes through the channels of stationary vanes (also called Guide Blades), only a portion of the available pressure difference (P_D-P_S) is converted into velocity

- Thus $C_3 < C_{3max\text{-}attainable}$ and, hence, the spec. work $Y_{blade} = U_2 C_{3U}$ of the reaction turbine is smaller that that of the impulse turbine if the same velocity U_2 is assumed

- ***The velocity U of reaction turbines has to be higher than that of impulse turbines if the same Y_{blade} is to be obtained.***

- **Reaction turbines may be classified as fast running turbomachines.**

- β_1 should be small but not too small as leads to strong whirls in the discharge flow.
- The angle β_2 of reaction turbines is $\beta_2 \leq 90^0$ and, thus, differs from that of impulse turbines.
- The blade does not have the hook form. As the relative velocity increases from W_3 to W_0, the channel width decreases and no profile is necessary in order to obtain equal channel width.
- Reaction turbine has more stages because of the lower Y_{blade} of its single stage.

Degree of Reaction

- In case of the reaction turbine, *the driving force at the rotor is due to change of direction (impulse) and magnitude (reaction) of the relative velocity W.*
- A reaction effect, i.e. *a change of the magnitude of the relative velocity W* in case of the axial-flow machine, is only possible *if a pressure difference exists between the entrance and discharge* side of the rotor.
- Quota of reaction acting is found by comparing total energy Y, with pressure rotor energy (Y_p).

$$Degree\ of\ reaction = \frac{Spec.\ Static\ rotor\ work}{Spec.\ work\ between\ inlet\ and\ outlet\ (of\ the\ stage)} = \frac{Y_P}{Y}$$

$$impulse\ machine: Y_P = 0 \quad and \quad R = 0$$

$$reaction\ machine: Y_P > 0 \quad and \quad 0 < R < 1 \qquad (R \geq 1\ in\ some\ special\ cases)$$

The reaction effect exists also in case of radial or mixed flow rotors where $U_1 \neq U_2$ even for $|W_0| = |W_3|$ as shown by the equation

$$Y_P = \frac{\frac{1}{2}\left(U_2^2 - U_1^2 + W_0^2 - W_3^2\right) \mp Z_u}{Y} = \frac{\frac{1}{2}\left(U_2^2 - U_1^2\right) \mp Z_u}{Y} \neq 0$$

Blade Speed Ratio

- The blade speed ratio as defined below is widely used in the calculation of turbines especially of steam turbines.

$$Blade\ Speed\ Ratio = \frac{U}{C_Y} = \frac{U}{\sqrt{2Y}}$$

- $C_Y = \sqrt{2Y}$ is the velocity which could be obtained if the spec. work Y is converted without losses completely into velocity.

$$C_Y \approx \frac{C_2}{\varphi\sqrt{1-R}}$$

Where Φ is velocity coefficient of guide vanes (referring to velocity losses)

After some derivation

$$\frac{U}{C_Y} = \frac{\eta_h}{2\varphi\cos\alpha_2}\frac{1}{\sqrt{1-R}}$$

- Assuming the following data: η_h = 0.85;ϕ=0.98; α_2= 30^0.

$$\frac{\eta_h}{2\varphi\cos\alpha_2} \approx 1$$

- The blade speed ratio has the value

$$\left(\frac{U}{C_Y}\right)_{R=0} = \frac{1}{2} \quad for \quad R=0$$

$$\left(\frac{U}{C_Y}\right)_{R=0.5} \approx \frac{1}{\sqrt{2}} \quad for \quad R=0.5$$

- The following values of the blade speed ratio re obtained with actual machines:

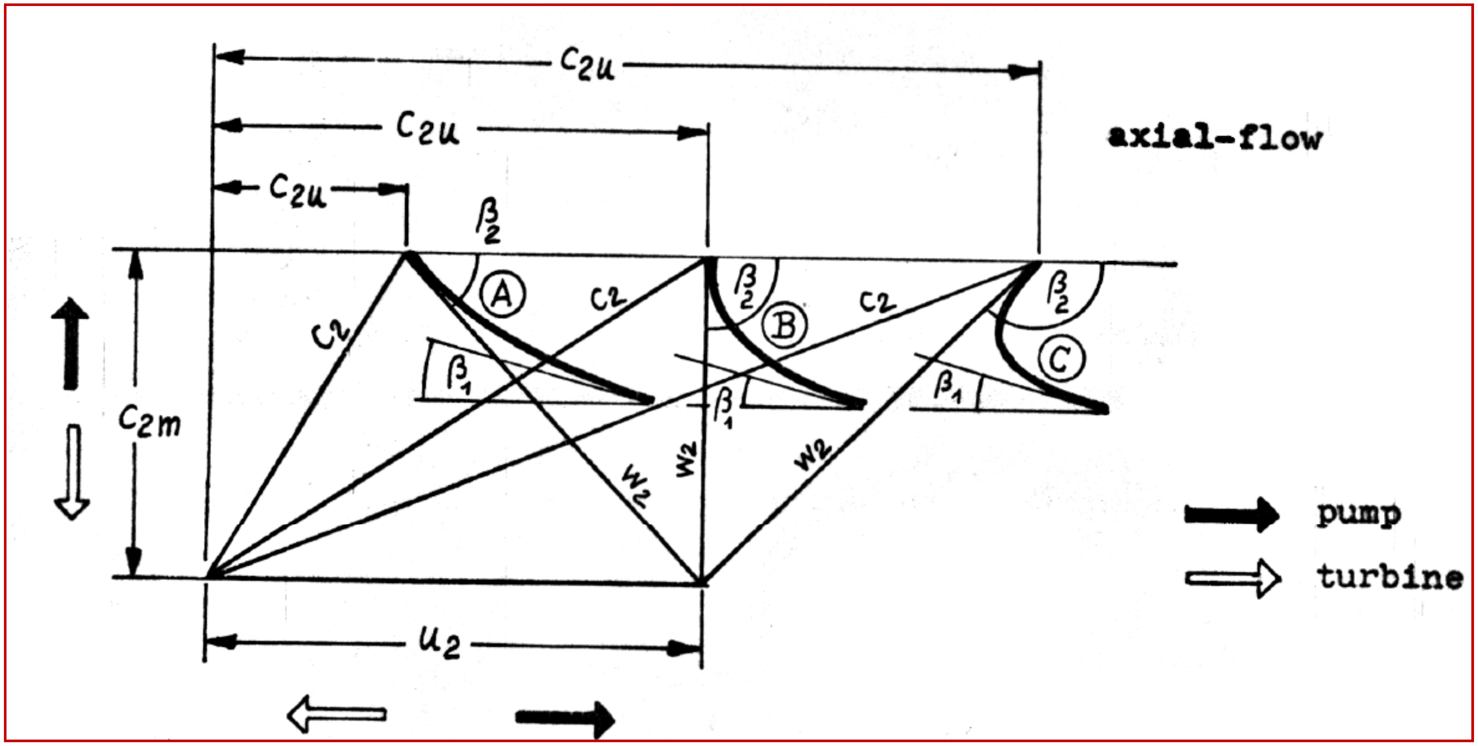
$$impuse\ steam\,turbines \left(\frac{U}{C_Y}\right)_{R=0} = \underbrace{0.35}_{Cheap\,Design.small\ power} \quad to \quad \underbrace{0.47}_{high\,quality\,design\ l\,arg\,e\ power} = k'$$

$$reaction\,steam\,turbines \left(\frac{U}{C_Y}\right)_{R>0} \approx \frac{k'}{\sqrt{1-R}} = \frac{0.35\,to\ 0.47}{\sqrt{1-R}}$$

$$Pelton\,Turbines \left(\frac{U}{C_Y}\right)_{R=0} = 0.44\ to\ 0.47$$

(57) Calculation of Mean Diameter and Peripheral Velocity using the Blade Speed Ratio

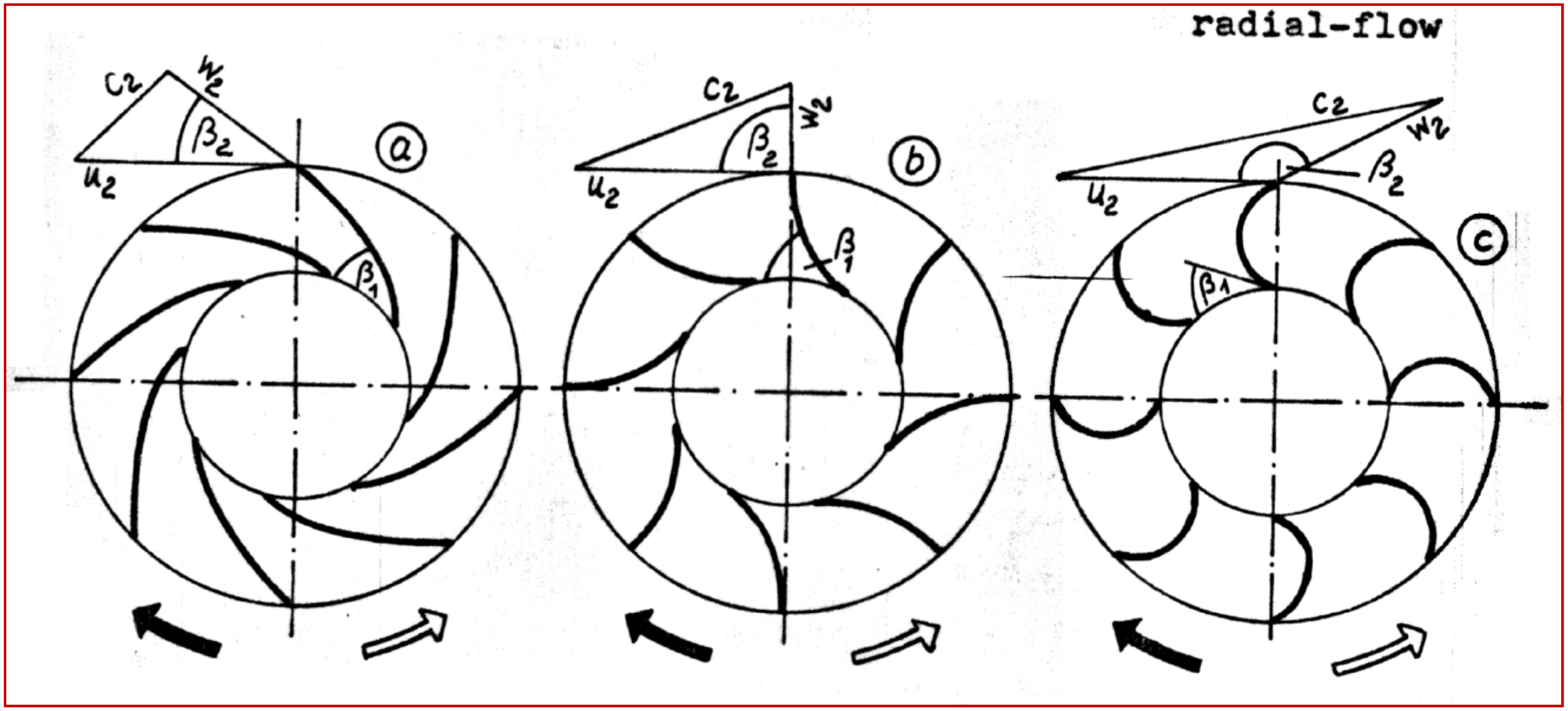
The Vane Angle β_2



- Three different axial-flow vanes, namely form A, B, C for which U_2, C_{2m} and β_1 are the same but the angle β_2 differ

- B_2 is chosen for the form A as $\beta_2<90^0$, for the form B as $\beta_2=90^0$ and for the form C as $\beta_2>90^0$.

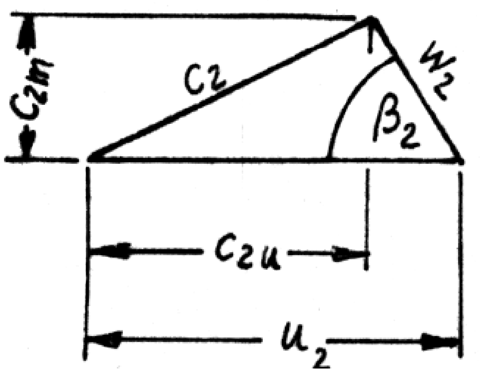
- A similar sketch for three different radial-flow vanes with $\beta_2<90^0$ (form a), $\beta_2=90^0$ (form b) and $\beta_2=90^0$ (form c) is given below.



- In case of the radial-flow machine the vanes of the different forms are called
  - Vane form a as ‘backward-curved’ vanes
  - Vanes form b, c as ‘forward-curved’ vanes

The following relation exists between β$_2$ and U$_2$

❖ **Case:α_0=90^0**



$$Y_{blade} = U_2 C_{3U} \quad and \quad Y_{blade\infty} = U_2 C_{2u}$$

$$where, C_{2u} = U_2 - W_{2U} = U_2 - C_{2m} \cot \beta_2, \quad then$$

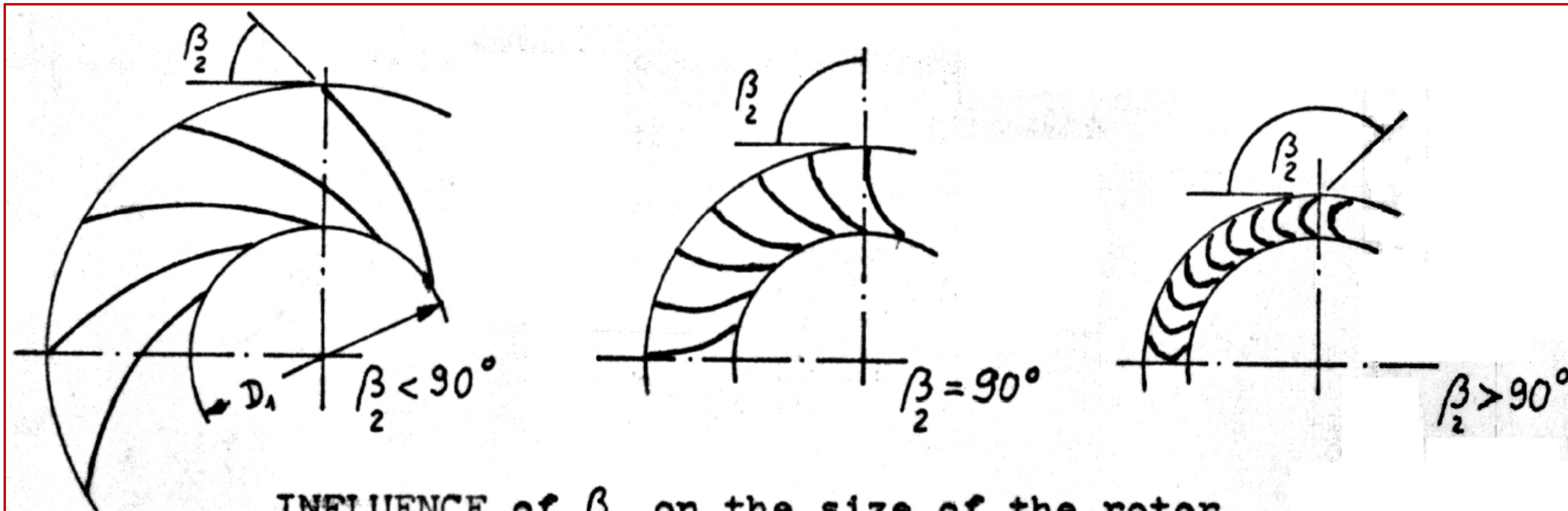
$$Y_{blade\infty} = U_2 (U_2 - C_{2m} \cot \beta_2) and\ it\ follows$$

$$U_2 = \frac{C_{2m}}{2 \tan \beta_2} + \sqrt{\left(\frac{C_{2m}}{2 \tan \beta_2}\right)^2 + Y_{blade\infty}}$$

❖ **Case:α_0≠90^0**

$$U_2 = \frac{C_{2m}}{2 \tan \beta_2} + \sqrt{\left(\frac{C_{2m}}{2 \tan \beta_2}\right)^2 + Y_{blade\infty} + U_1 C_{OU}}$$

- The necessary peripheral velocity U_2 for a given $Y_{blade\infty}$ can be determined by these equation if the vane angle β_2 is assumed.
- A large β_2 decreases U_2 and the size of the rotor decreases, too, if the speed n is not altered:



INFLUENCE of β_2 on the size of the rotor

(speed n, D_1 and β_1 assumed to be same)

Ranges of β_2 with different types of turbomachines

Shape Number, Specific Speed

- The shape of the rotor is determined by the three related values n, Y and V as long as the vane angle β_2 is unchanged.

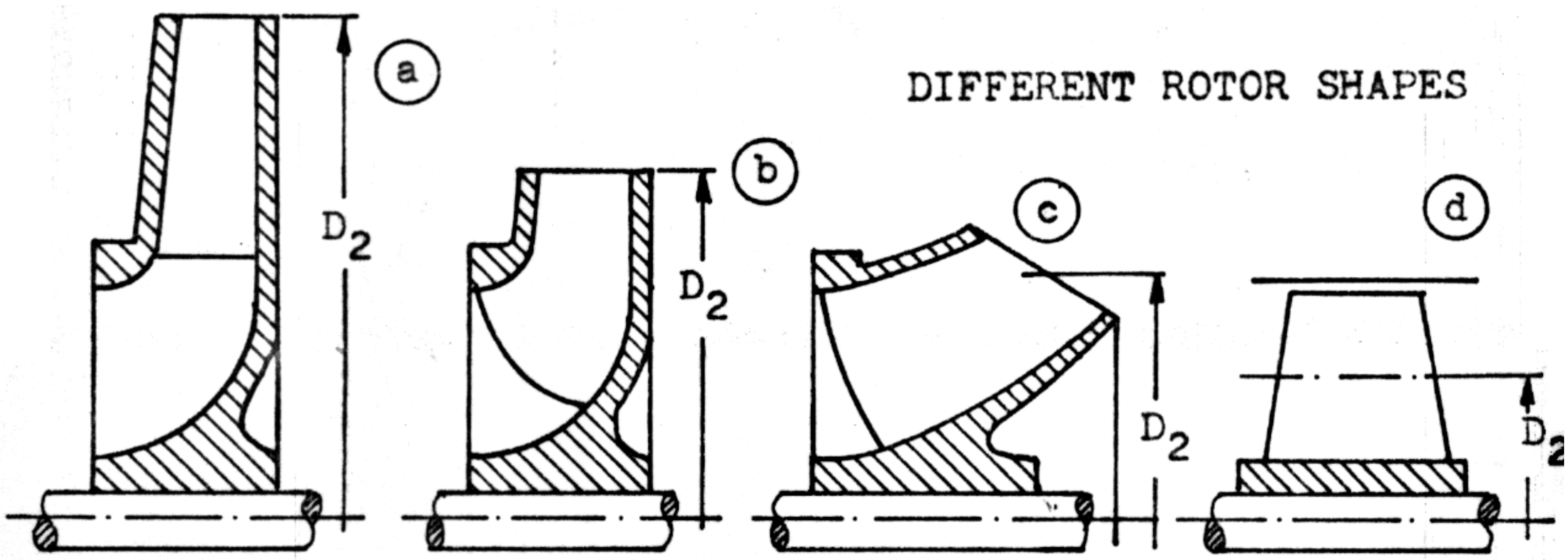
1. **Effect of Increase in speed n on the shape of the rotor (with unchanged β_2,V and Y)**

❖ The unchanged Y demands the same velocity triangle at 2.

$$Y \propto Y_{blade} \propto Y_{blade\infty} = U_2 C_{2U}$$

❖ The unchanged velocity triangle can be obtained for ***increased speed n but same velocity U as demanded by the unchanged velocity triangle only at a smaller outer diam***.

DIFFERENT ROTOR SHAPES



2. Effect of Increase in V on the shape of the slow running rotor (with unchanged β_2,n ,D_2,and Y)

- The larger volume V can be obtained only by **increasing the channel width and the eye dia. Ds**
- The meridian component of the velocity must remain unchanged because of the unchanged Y with same n and D_2
- Demanding and unchanged velocity triangle at 2.

- The rotor shape is a function of n, V and Y.
- **Shape number (N_{shape})** is a dimensionless number and is used to define the shape of the rotor by relating n, V and Y.

$$N_{shape}[1]=\left[n\frac{1}{s}\right]^{\alpha}\left[V\frac{m^3}{s}\right]^{\beta}\left[Y\frac{m^2}{s^2}\right]^{\gamma}, assume\ \alpha=1;$$

$$1=\left[\frac{1}{s}\right]^{1}\left[\frac{m^3}{s}\right]^{\beta}\left[\frac{m^2}{s^2}\right]^{\gamma}=m^0s^0$$

- It follows

$$\begin{aligned}
&m: && 3\beta+2\gamma=0 \\
&S: && \underline{-1-\beta-2\gamma=0} \\
& && -1+2\beta=0 && or && \beta=\frac{1}{2} \\
&thus && 2\gamma=-\frac{3}{2} && or && \gamma=-\frac{3}{4}
\end{aligned}$$

Thus,

$$N_{shape}[1]=n^1V^{1/2}Y^{3/4}=\frac{n\sqrt{V}}{Y^{3/4}}$$

$$n_{sh}=1000N_{shape}$$

- A relation which is based on the head H instead on the spec. work Y is called ***Specific Speed.***

$$n_q = \frac{n\sqrt{V}}{H^{3/4}}$$

- Where the values has a unit of n(rpm), V(m^3/s) and H(m).
- n_q is not dimensionless for metric system n_q has the following unit

$$n_q = 9.81^{3/4}\left[\frac{m}{s^2}\right]^{3/4}\frac{60s}{1\min}N_{shape} = 333N_{shape}\left[\frac{m^{3/4}}{\sqrt{s}.\min}\right]$$

- For water turbines a specific speed derived from n, H and N is often used.

$$n_s = \frac{n\sqrt{N}}{H^{5/4}}$$

Values of N_{shape}, n_q and n_s:			
	$1000N_{shape}$	n_q	n_s
Slow- running rotor	33 to 120	11 to 38	40 to 140
Medium-running rotor	120 to 250	38 to 82	140 to 300
Fast –running rotor	250 to 500	82 to 164	300 to 600
axial-flow rotor	330 to 1500	110 to 500	400 to 1800